Critical heat current for operating an entanglement engine

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Two-qubit entanglement engine

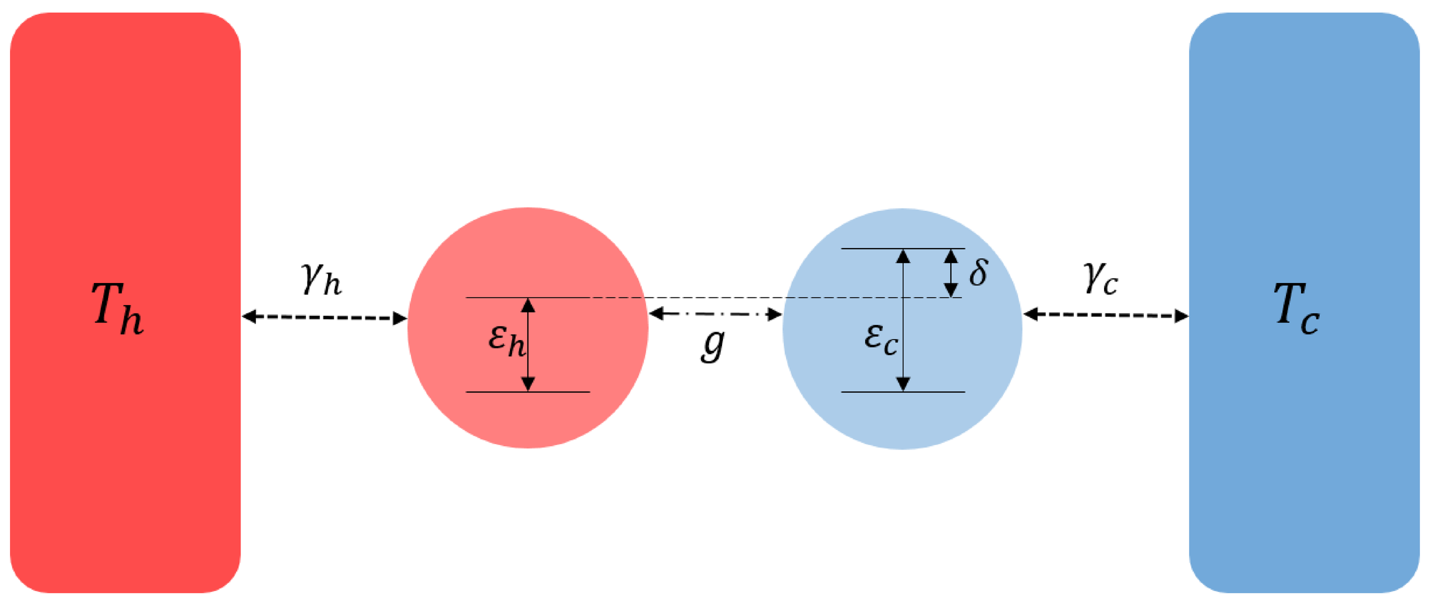

Fig. 1: Two-qubit thermal machine

$$H = H_{\mathsf{S}} + H_{\mathsf{int}} + H_{\mathsf{B}} + H_{\mathsf{SB}} \tag{1}$$

Local master equation

The local Lindblad equation (valid for weak-inter-qubit coupling [2]) for the machine is given by

$$\dot{\rho}(t) = -i[H_{\mathsf{S}} + H_{\mathsf{int}}, \rho(t)] + \sum_{j\in\{h,c\}} \gamma_j^+ \mathcal{D}\left[\sigma_+^{(j)}\right]\rho(t) + \gamma_j^- \mathcal{D}\left[\sigma_-^{(j)}\right]\rho(t) \tag{2}$$

with the dissipators $\mathcal{D}[A]\rho(t) := A\rho(t)A^\dagger - \{A^\dagger A, \rho(t)\}/2$. For Bosonic baths, $\gamma_j^+ = \gamma_j n_B^{(j)}$ and $\gamma_j^- = \gamma_j(1 + n_B^{(j)}), \quad j \in \{h, c\}$.

$$\rho_{\mathsf{ss}} = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & c & 0 \\ 0 & c^* & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix}. \tag{3}$$

The steady-state heat current is defined as $J_{\mathsf{ss}} := Q_{\mathsf{h}}^{\mathsf{ss}} - Q_{\mathsf{h}}^{\mathsf{ss}}$

$$Q_j(t) = \mathsf{Tr}\left[H_{\mathsf{S}}\left(\gamma_j^+\mathcal{D}_j\left[\sigma_+^{(j)}\right]\rho(t) + \gamma_j^-\mathcal{D}_j\left[\sigma_-^{(j)}\right]\rho(t)\right)\right]. \tag{4}$$

The steady-state negativity is given by

$$N(\rho_{\mathsf{ss}}) = \max\{0, n(\rho_{\mathsf{ss}})\}, \tag{5}$$

where

$$n(\rho_{\mathsf{ss}}) = \frac{1}{2}\left(\sqrt{4|c|^2 + (r_1 - r_4)^2} - (r_1 + r_4)\right). \tag{6}$$

Using the exact form of the steady state,

$$c = \frac{J_{\mathsf{ss}}(2\delta - i\Gamma)}{4g(\varepsilon_h\Gamma_c + \varepsilon_c\Gamma_h)}, \tag{7}$$

Combining with the condition for non-zero steady-state entanglement, $|c|^2 > r_1 r_4$ (PPT criterion), one obtains a lower bound on heat current for non-zero entanglement in the steady state.

$$J_{\mathsf{ss}} > \sqrt{\frac{16g^2 r_1 r_4}{\Gamma^2 + 4\delta^2}(\varepsilon_h\Gamma_c + \varepsilon_c\Gamma_h)^2} := J_{\mathsf{c}} \tag{8}$$

Necessary and sufficient condition for the engine to operate successfully!

Results

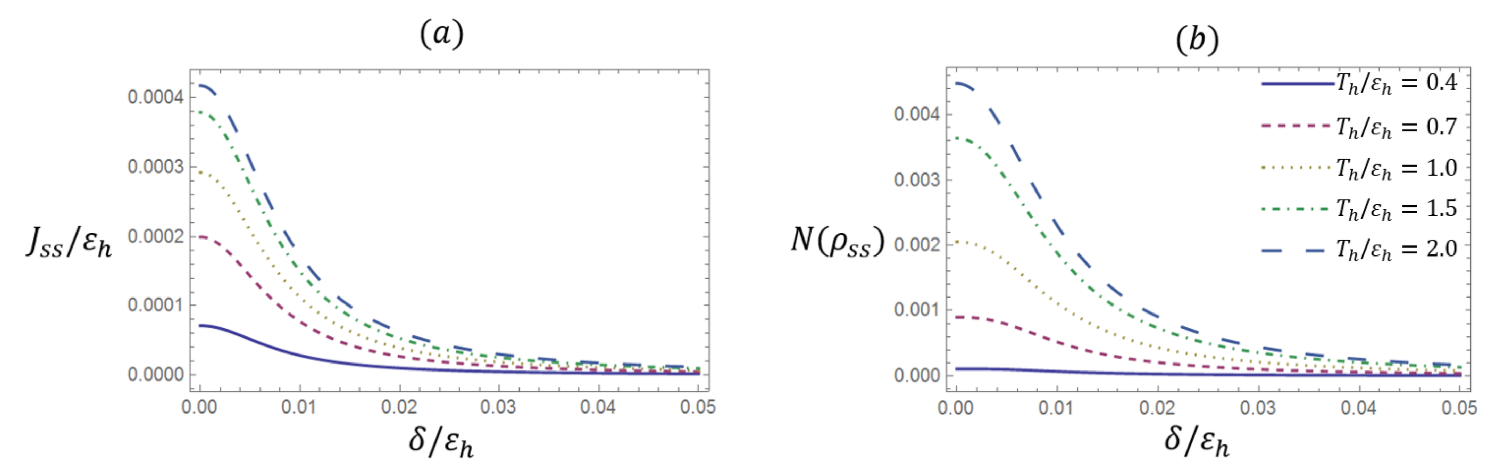

Fig. 2: Steady-state (a) heat current J_{ss}/ε_h and (b) negativity $N(\rho_{ss})$ as functions of κ_h/ε_h and δ/ε_h, with $\kappa_c = 0$, $T_h/\varepsilon_h = 0.7$, $T_c/\varepsilon_h = 0.1$, $g/\varepsilon_h = 1.6\times10^{-3}$, $\gamma_h/\varepsilon_h = 10^{-3}$ and $\gamma_c/\varepsilon_h = 1.1\times10^{-2}$.

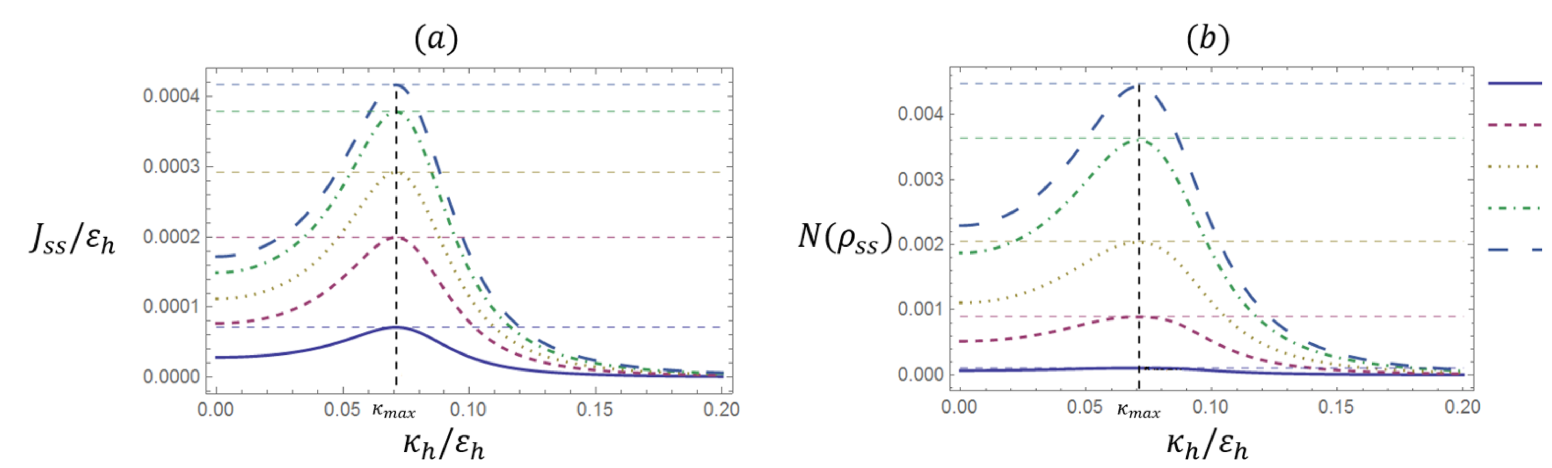

Fig. 3: Steady-state heat current J_{ss}/ε_h and negativity $N(\rho_{ss})$ as functions of κ_h/ε_h, with $\kappa_c = 0$ and $\delta/\varepsilon_h = 0.01$. The horizontal dashed curves mark the heat current and negativity respectively, for $\delta = 0$ and $\kappa_h = \kappa_c = 0$, for different values of T_h/ε_h. $T_c/\varepsilon_h = 0.1$, $g/\varepsilon_h = 1.6\times10^{-3}$, $\gamma_h/\varepsilon_h = 10^{-3}$ and $\gamma_c/\varepsilon_h = 1.1\times10^{-2}$

Critical heat current

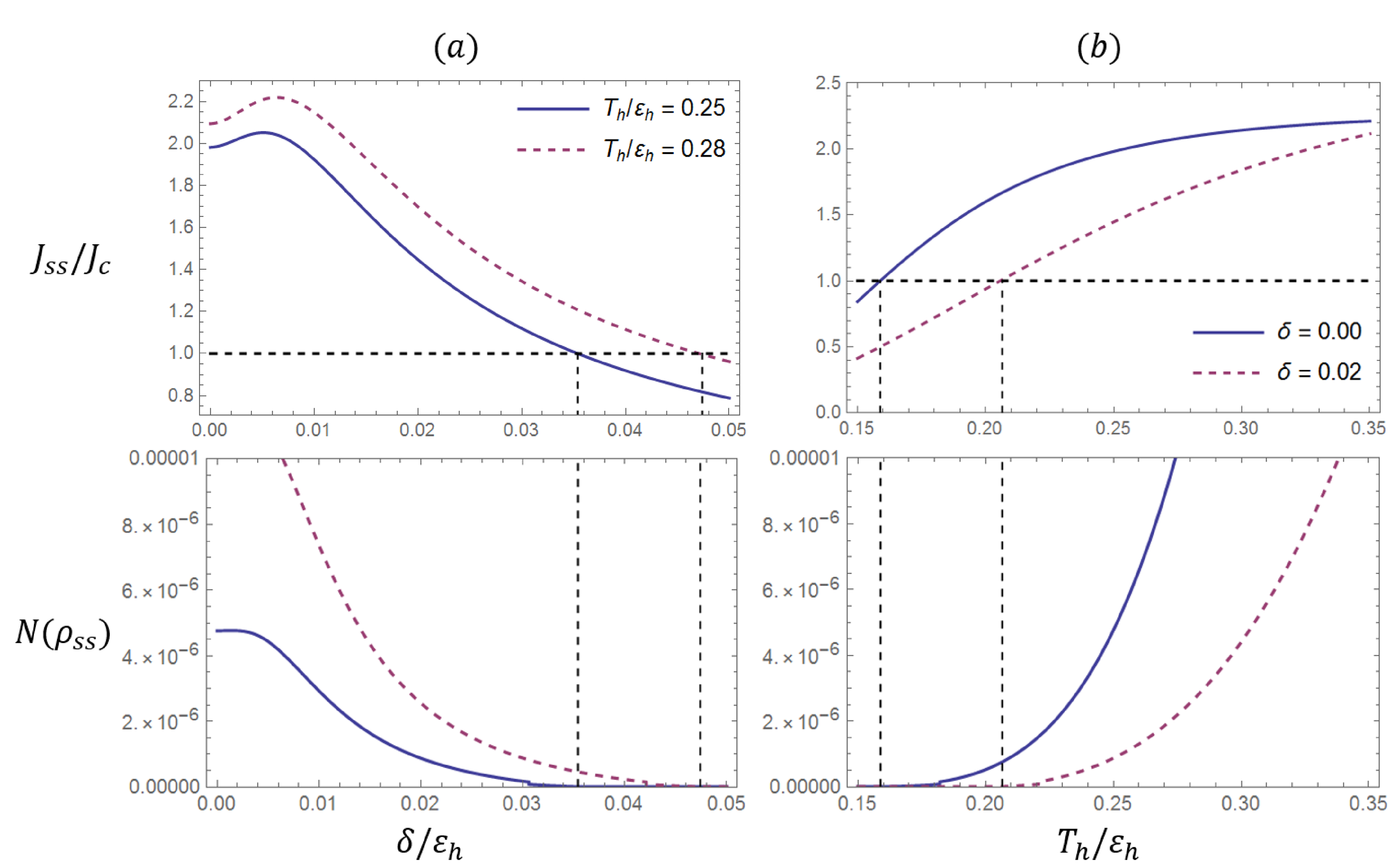

Fig. 4: Steady-state (a) heat current J_{ss}/ε_h and (b) negativity $N(\rho_{ss})$ as functions of κ_h/ε_h and δ/ε_h, with $\kappa_c = 0$, $T_h/\varepsilon_h = 0.7$, $T_c/\varepsilon_h = 0.1$, $g/\varepsilon_h = 1.6\times10^{-3}$, $\gamma_h/\varepsilon_h = 10^{-3}$ and $\gamma_c/\varepsilon_h = 1.1\times10^{-2}$.

Global master equation

For strong-inter-qubit coupling, we rely on a global master equation [2]

$$\dot{\rho}(t) = -i[H_{\mathsf{S}} + H_{\mathsf{int}}, \rho(t)] + \sum_{\alpha\in\{-,+\}}\sum_{j\in\{h,c\}} \gamma_j^-(\varepsilon_\alpha)\mathcal{D}\left[\hat{L}_j(\varepsilon_\alpha)\right]\rho(t) + \gamma_j^+(\varepsilon_\alpha)\mathcal{D}\left[\hat{L}_j^\dagger(\varepsilon_\alpha)\right]\rho(t) \tag{9}$$

$$\begin{aligned} \hat{L}_j(\varepsilon_-) &= |0\rangle\langle 0|\,\sigma_-^{(j)}\,|\varepsilon_-\rangle\langle\varepsilon_-| + |\varepsilon_+\rangle\langle\varepsilon_+|\,\sigma_-^{(j)}\,|2\rangle\langle 2| \\ \hat{L}_j(\varepsilon_+) &= |0\rangle\langle 0|\,\sigma_-^{(j)}\,|\varepsilon_+\rangle\langle\varepsilon_+| + |\varepsilon_-\rangle\langle\varepsilon_-|\,\sigma_-^{(j)}\,|2\rangle\langle 2|, \end{aligned} \tag{10}$$

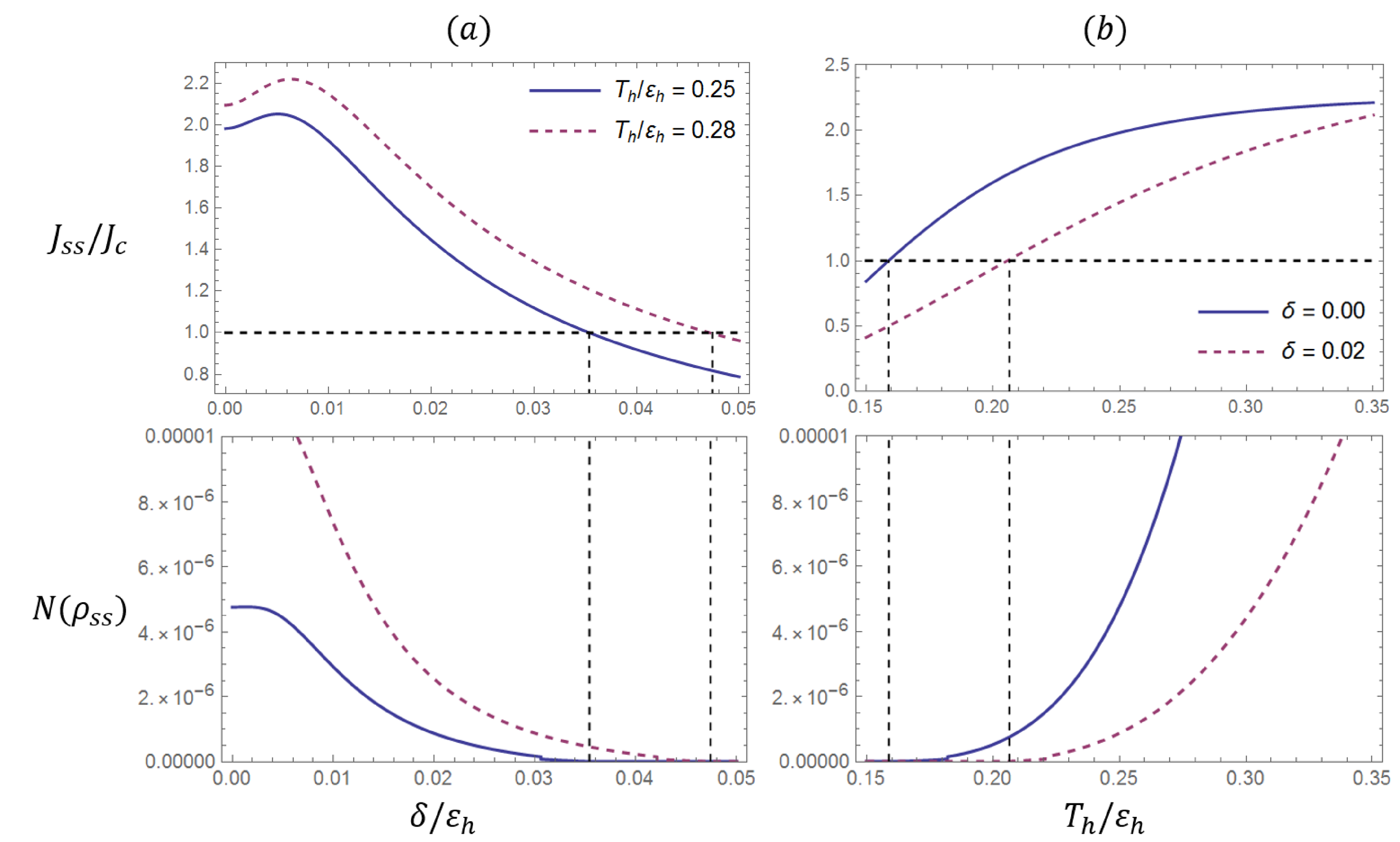

Fig. 5: (a)Steady-state negativity $N(\rho_{ss}^{gl})$ as functions of temperature T_h, for $T_c/\varepsilon = 0.01$, $\gamma_h/\varepsilon = 0.01$, $\varepsilon = 1$ and different values of g.

(b) $N(\rho_{ss}^{gl})$ as a function of g, with $T_c/\varepsilon = 0.01$, $\gamma_h/\varepsilon = 0.01$, $\varepsilon = 1$ and different values of γ_c. The value of T_h is optimised to give the maximum negativity.

Conclusions

- To find non-zero entanglement in the steady state, there exists a critical steady-state heat current which needs to be maintained.
- There is a critical heat current in both global and local approaches to the master equation.
- Strong inter-qubit coupling is not superior to weak-inter qubit coupling and involves the problem of entanglement at thermal equilibrium.

References

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[2] P.P. Hofer, M. Perarnau-Llobet, L.D.M. Miranda, G. Haack, R. Silva, J.B. Brask and N. Brunner. New Journal Physics, **19** 123037 (2017)

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